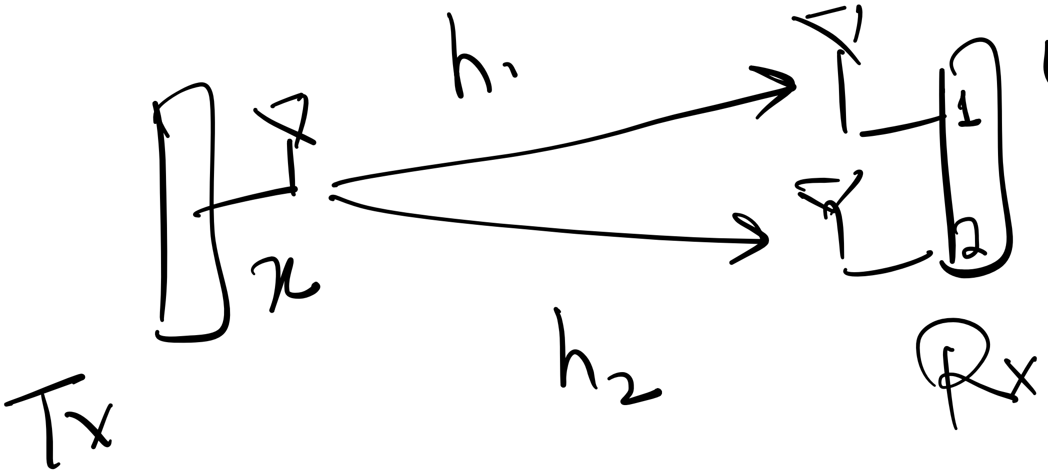
Multi-antenna Systems

- Multiple receive antennas
- Multiple transmit antennas
- MIMO
- Nulling ← [Interference Management]
- Alignment.

Receiver Diversity



$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

① Decode both streams and compare.

② Add these two received signals.

$$y_1 + y_2 = (h_1 + h_2)\, x + (n_1 + n_2)$$

complex

high risk

(+1, +1)

aligned → strong

(+1, -1)

anti-aligned → weak

$$\left| \boxed{\alpha_1}\, \underset{\overset{\uparrow}{h_1}}{\underline{y_1}} + \boxed{\alpha_2}\, \underset{h_2}{\underline{y_2}} \right|$$ is maximized

Maximal ratio combining

$\alpha_1 = \underline{h_1}^*$ $\qquad$ $\alpha_2 = h_2^*$

↳ make the phase negative.

$$\alpha_1 y_1 = h_1^* h_1 x_{\cancel{k}} + h_1 n_1$$

$$= |h_1|^2 x_{\cancel{k}} + h_1^* n_1$$

$$\alpha_2 y_2 = h_2^* h_2 x_{\cancel{k}} + h_2^* n_2$$

$$\alpha_1 y_1 + \alpha_2 y_2 = \left(|h_1|^2 + |h_2|^2\right) x$$
$$+ h_2^* n_2 + h_1^* n_1$$

$$SNR = \frac{S}{N} = \frac{\left(|h_1|^2 + |h_2|^2\right)^2 |x|^2}{E\left[|h_2^* n_2 + h_1^* n_1|^2\right.}$$

Before: $\dfrac{|h_1|^2 x^2}{n_1^2}$

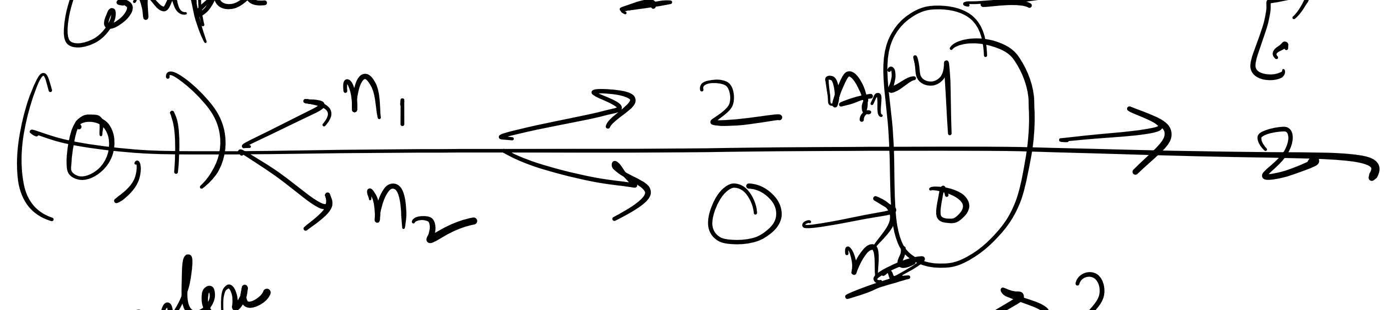
$$\text{random} \rightarrow \sigma \text{ std.}$$

After

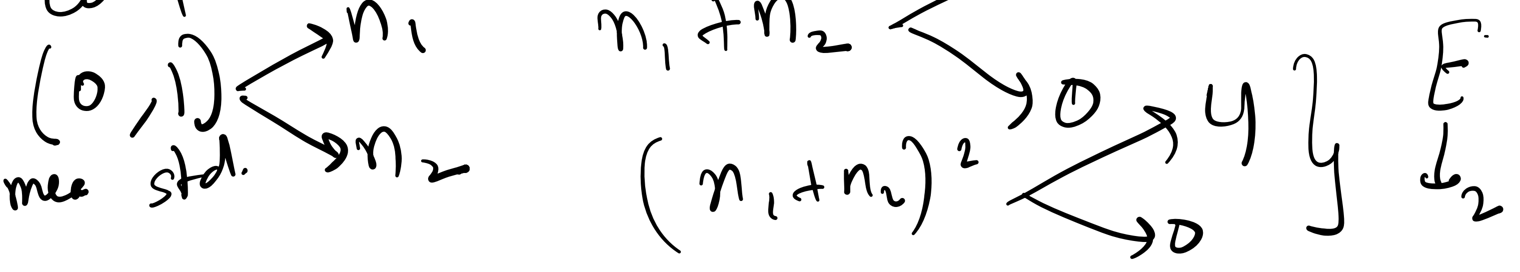
$$E\left[\left|\overset{\text{random}}{h_1^*}\, n_1 + \overset{\text{random}}{h_2^*}\, n_2\right|^2\right]$$

$$= \left(|h_1|^2 + |h_2|^2\right) \sigma^2$$

Comple



complex



$$\mathrm{SNR}_{after} = \frac{\left(|h_1|^2 + |h_2|^2\right) x^2}{\cancel{(|h_1|^2 + |h_2|^2)}\; n^2}$$

$$= \frac{\left(|h_1|^2 + |h_2|^2\right) x^2}{n^2}$$

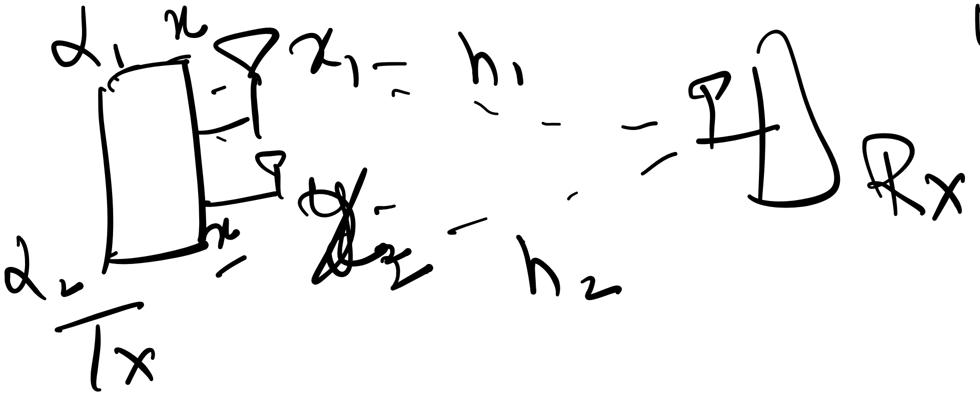
$$\cancel{S_{NR}}\ \frac{\mathrm{SNR}_{after}}{\mathrm{SNR}_{before}} = \frac{|\overset{\to 0.1}{h_1}|^2 + |\overset{10}{h_2}|^2}{|h_1|^2}$$

① $|h_1|^2 \simeq |h_2|^2$; 2

② $|h_1|^2 >> |h_2|^2$, not much advantage

③ $|h_1|^2 << |h_2|^2$, ↑ lot of advantage

Transmitter Diversity



$$y = h_1 x_1 + h_2 x_2 + n$$

Options:

↳ $x_1 = x_2 = x$

$$y = (h_1 + h_2) x + n.$$

$$y = \cancel{\alpha_1 x}\ \alpha_1 h_1 x + \alpha_2 h_2 x + n$$

$$= \underbrace{(\alpha_1 h_1 + \alpha_2 h_2)} x + n$$

$\alpha_1 = h_1^*$ $\alpha_2 = h_2^*$

$$y = (|h_1|^2 + |h_2|^2) x + n$$

"precoding"

wavelength

Rule of Thumb → $\lambda/2$ away → different channels

$$\alpha_1 = \frac{h_1^*}{\sqrt{|h_1|^2+|h_2|^2}} \qquad \alpha_2 = \frac{h_2^*}{\sqrt{|h_1|^2+|h_2|^*}}$$

normalization

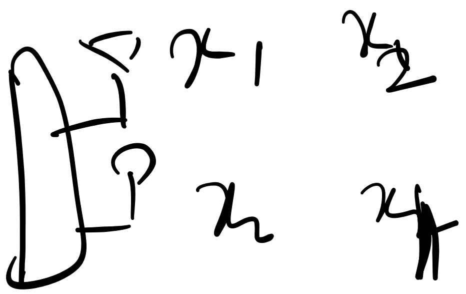
$$\alpha_1^2 + \alpha_2^2 = 1$$

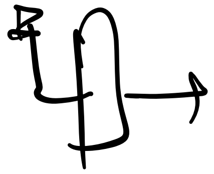
$$y = \left(\frac{|h_1|^2 + |h_2|^2}{\sqrt{|h_1|^2+|h_2|^2}}\right) x + n$$

$$SNR = \left(\frac{(|h_1|^2 + |h_2|^2)^2}{\cancel{(|h_1|^2 + |h_2|^2)}}\right) \frac{x^2}{n^2}$$

$$\frac{SNR_{after}}{SNR_{before}} = \frac{|h_1|^2 + |h_2|^2}{|h_1|^2}$$

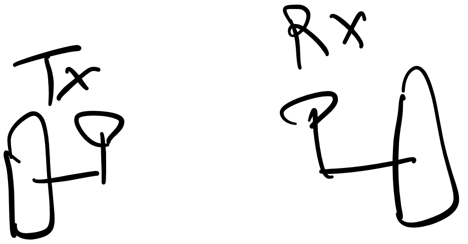
$$\boxed{\text{Transmitter needs to know the channel.}}$$

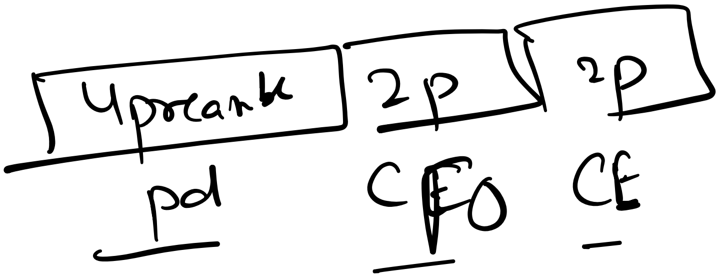


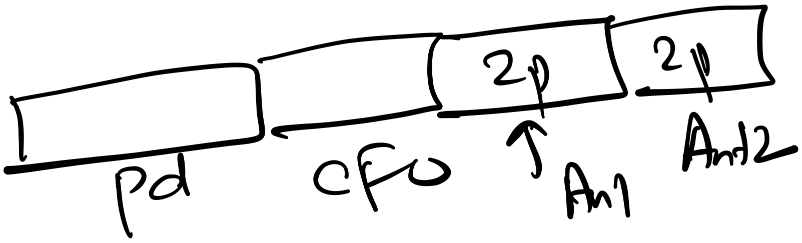


$$y = (h_1 x_1 + h_2 x_2) + n$$

$$y = (h_1 x_2 + h_2 x_1) + n$$



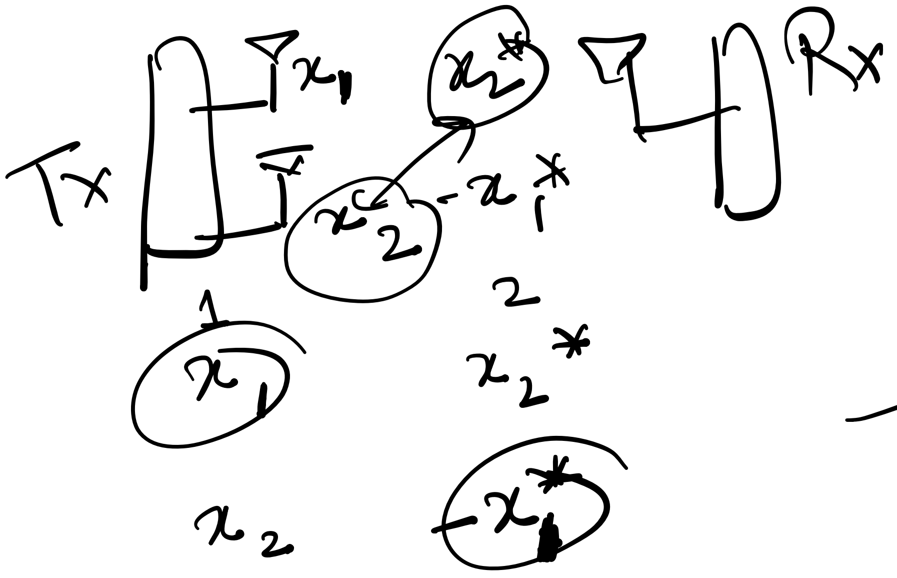




Space-time Codes

Alamouti

"I don't want channel feedback"



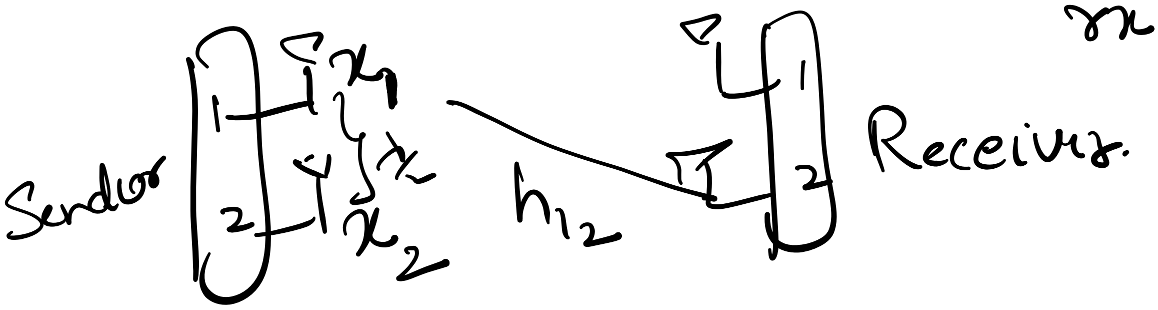
→ spread out the same info in space & time

$$\left.\begin{aligned} y_1 &= h_1 \underline{x_1} + h_2 \underline{x_2} + n \\ y_2 &= h_1 \underline{x_2^*} - h_2 \underline{x_1^*} + n \end{aligned}\right]$$

$$\underline{h_1^*}\,\underline{y_1} - \underline{h_2}\,\underline{y_2^*} = \boxed{h_1^* h_1 x_1} + \cancel{h_1^* h_2 x_2} - \cancel{h_2 h_1^* x_2} + \boxed{h_2^* h_2 x}$$

MIMO

h_{ij} ← channel from tx ant i to rx ant j



$$y_1 = h_{11} x_1 + h_{21} x_2 + n_1$$

$$y_2 = h_{12} x_1 + h_{22} x_2 + n_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\vec{y} = H \vec{x} + \vec{n}$$

↑ need to know this.

↑ Know this

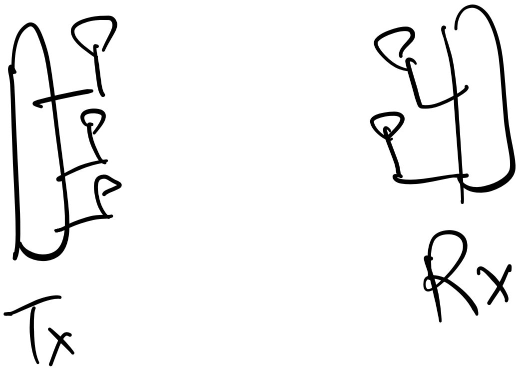
$$H^{-1}\vec{y} = H^{-1} H \vec{x} + H^{-1}\vec{n}$$

$$H^{-1}\vec{y} = \vec{x} + H^{-1}\vec{n}$$

H^{-1} to be reasonable, H to be inverstible.

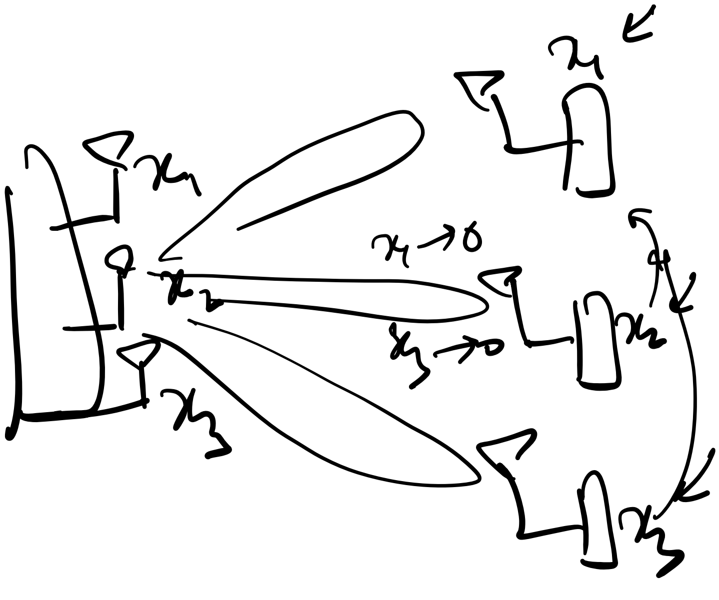
$$\underset{5\times1}{\vec{y}} = \underset{5\times5}{H}\,\vec{x} + \underset{5\times1}{\vec{n}}$$

$$\vec{y}_{5\times1} = H_{5\times2}\,\vec{x}_{2\times1} + \vec{n}$$



2 data streams at the same time

smaller number of antennas.



$$y_1 = h_{11}x_1 + h_{21}x_2 + h_{33}x_3$$

$$y_2 = h_{12}x_1 + h_{22}x_2 + h_{32}x_3$$

$$y_3 = h_{13}x_1 + h_{23}x_2 + h_{33}x_3$$

$$\vec{y}_{3\times1} = H_{3\times3}\,\bar{x}_{3\times1} + \bar{n}$$

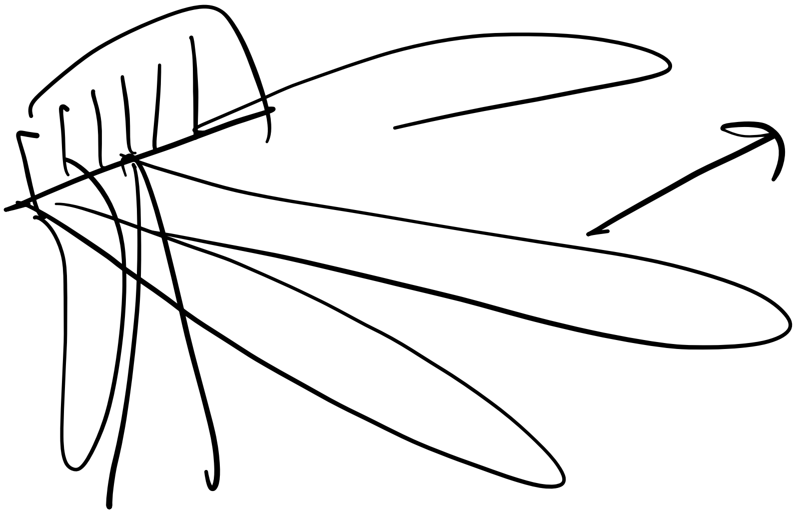
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \text{Send} \rightarrow \underbrace{H^{-1}\,\vec{x}} = H^{-1}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

[Zero-forcing]

$$\vec{y} = \underbrace{H\,H^{-1}}\,\bar{x} + n$$

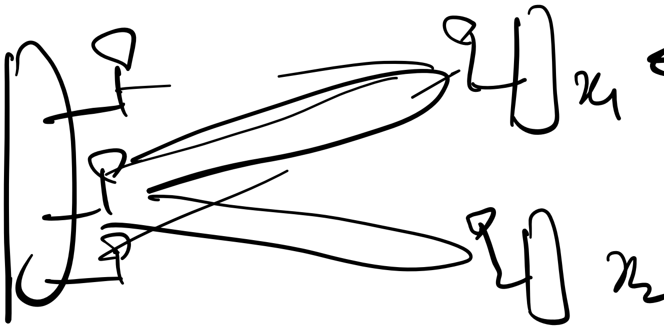
$$\bar{y} = \bar{x} + n \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{bmatrix} x_1 \\ \enclose{circle}{x_2} \\ \enclose{circle}{x_3} \end{bmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$



→ Multi-user MIMO

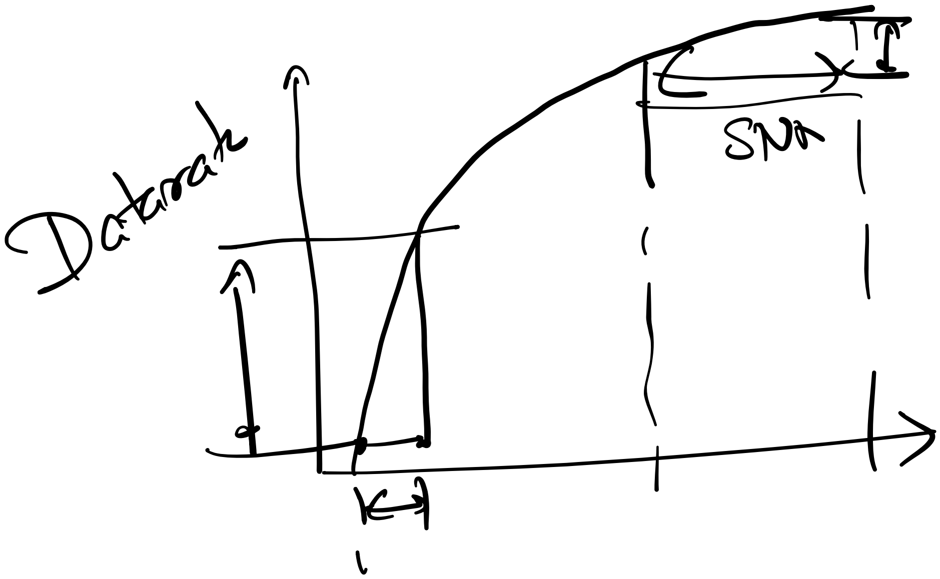
Diversity vs Multiplexing



← improve SNR
diversity //

multiplexing

$\text{Datarate} \propto \underline{BW} \log(\underline{\underline{SNR}})$



advantage is small.

low SNR → diversity, SNR↑,
datarate↑↑

high SNR → multiplexing, 2X the
<u>throughput</u>

Rate adaptation.

MCS table

↳ Modulation and coding
spatial stream)

0	BPSK	1
1	⋮	1
2	⋮	1
⋮	⋮	1
⋮	16-QAM	1
⋮	BPSK	2
⋮	⋮	3
32		4

Nulling

Alignment